GEAR TRAINS

or "How To Design Compound Gearing To Give Specific Ratios"

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This feature is based on an article which appeared in a recent North Midlands Meccano Group Newsletter, copies of which are obtainable from the N.M.M.G. Secretary at the address given in 'Meccano Club Roundup'. Our thanks go to the Editor for his kind permission to reprint.

THE BACKGROUND TO THE PROBLEM

It is well known that the design of a gear train which will give a specific overall gear ratio can often be a complex exercise, especially when the desired ratio is one of those 'odd' values so often found in astronomical mechanisms. However such design problems can be eased by the inclusion of a differential in the system, since a differential is, in fact, merely a convenient gear assembly in which the cage rotates at the average speed of the two half-shafts. Thus the achievement of a difficult gear ratio may well be made possible by the use of a layout such as indicated in Figure 1, where the input drive is split into two parts, drive A being taken to one half-shaft and drive B to the differential's cage, with in each case a gear train (C_A or C_B) being inserted before the differential is reached. The output is taken from the second half-shaft, and the resultant effect is such that:

$$\frac{\text{Output Speed}}{\text{Input Speed}} = \frac{2}{b} - \frac{1}{a}$$

where a and b are the reduction ratios of gear trains C_A and C_B respectively (that is, for gear train C_A (output speed/input speed) = 1/a, and similarly for C_B).

Such a procedure can, of course, be extended to include the use of multiple differentials arranged in either series or parallel, with intermediate gear trains as appropriate, and a very high level of accuracy can be achieved. The differential unit itself can be built up in the form of any one of the wide range of conventional types which use an inner gear set of either bevels or pinions and contrates, or alternatively an all-pinion layout can be used. This could be either a full set of pinions as is usual in a spur differential, or could be just a half set with the output then being taken from a pair of universal couplings as was described by Alan Partridge in Meccano Magazine 1977 No. 2. These various possibilities are sketched in Figure 2; the fact is that the inner workings of the differential are unimportant, the unit being used solely for its capabilities as a shaft-speed averaging device.

A NEW APPROACH

A considerable further development from this technique can be made by modifying the spur-to-universal coupling type of differential and at the same time allowing the total gear train layout to be inverted so that the slave gear trains (C_A and C_B of Figure 1) follow the differential rather then precede it, and link the half-shafts rather than a half-shaft and the cage. The general layout is then as indicated in Figure 3. The input is now applied only to the cage of the unit containing two gears A and B, which are not necessarily the same size as each other. These gears are mounted on half-shafts X and Y which are then linked to the output shaft by gear trains C_X and C_Y having reduction ratios x and y respectively.

If A and B are, in fact, the numbers of teeth on their respective gears, the resultant effect of the complete layout is such that

$$\frac{\text{Output Speed}}{\text{Input Speed}} = \frac{A + B}{Ax + By}$$

The derivation of this formula is given in the original N.M.M.G. Newsletter version of this article for those interested in such matters; the major point, however, is that the simplicity and flexibility of the expression makes it a very powerful tool in gearing design.

APPLICATIONS

EXAMPLE A One of the great problems in Meccano gearings is the handling of ratios in which the numbers involved are powers of 2 and 3, where it is easy to get into a wasteful gear train of alternating step-up and step-down stages. For instance, in clocks and orreries where it is desired to have a shaft revolving once per lunar month (or Lunar Synodic Period as it is formally known) a reduction factor of 1 : 29.530589 from the once-per-day shaft must be provided. The closest approximation directly available with numbers interpretable in Meccano gears is 32/945 = 1 : 29.531250 giving an error of 661×10^{-6}. This ratio can be broken down to 19/133 x 19/95 x 32/27 for Meccano usage, the first two ratios then being immediately available with standard gears, but leaving the 32/27 as the problem. Either a combination of step-up and step-down gearing has to be used, or alternatively the (A+B)/(Ax+By) method makes it simple;

i) $\frac{32}{27} = \frac{20 + 12}{20 + 7} = \frac{A + B}{Ax + By}$

ii) Thus A/B = 20/12 which in Meccano could become 25/15 as a result of multiplying each term in the expression by 5/4

iii) 1/x = A/Ax = 20/20 = 1 – this means that the left-hand half-shaft rotates at the same speed, and in the same direction, as the output shaft (and thus might just as well actually *be* the output shaft)

iv) 1/y = B/By = 12/7 which is achievable in Meccano by a layout as sketched in Figure 4 (note that $ indicates a sprocket wheel).

Note that the 19-tooth Pinion between the 57-tooth and 38-tooth Gears is merely an idler; if such an idler were not used, the two shafts, being linked, would rotate in opposite directions,

Fig. 1.

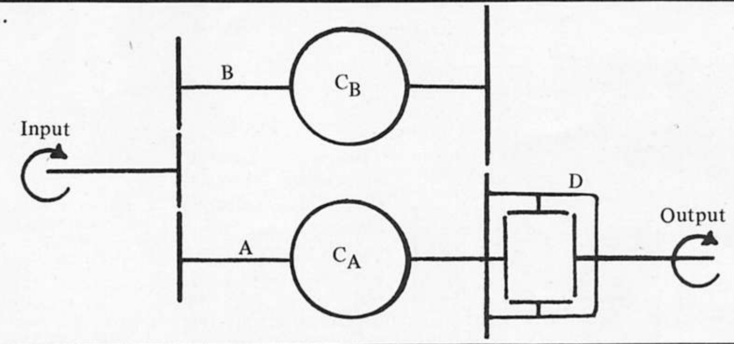
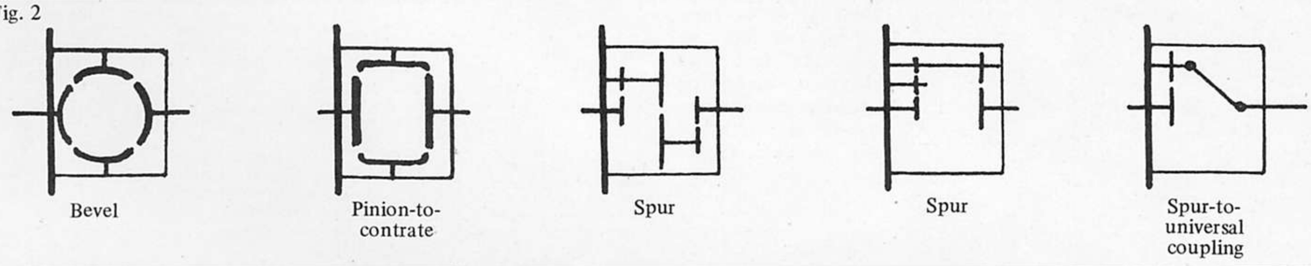


Fig. 2



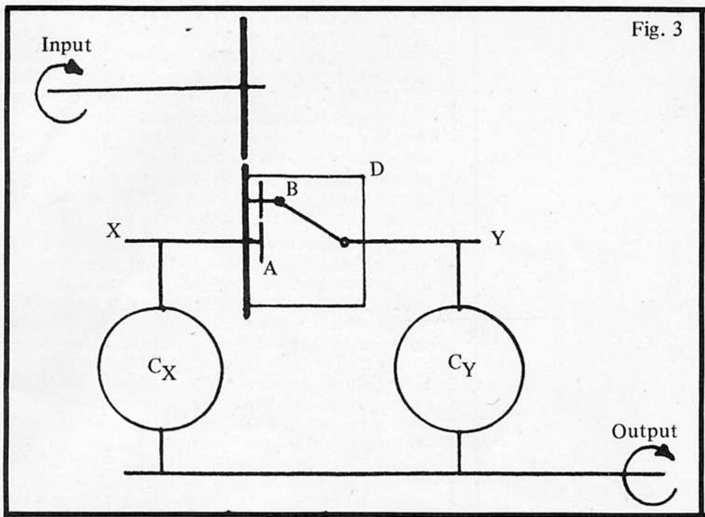


Fig. 3

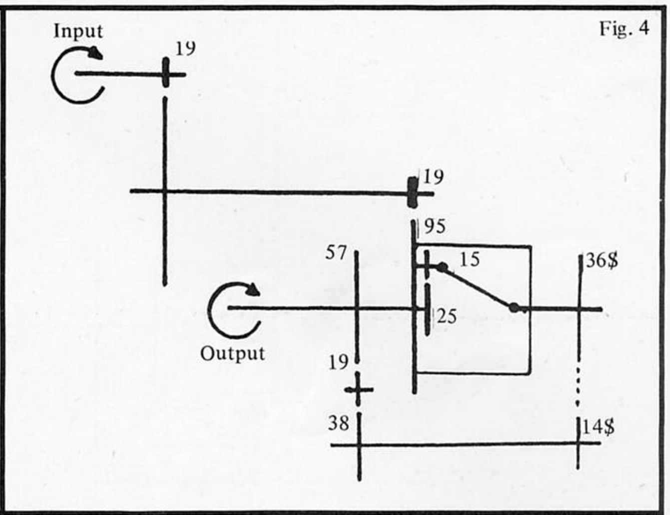


Fig. 4

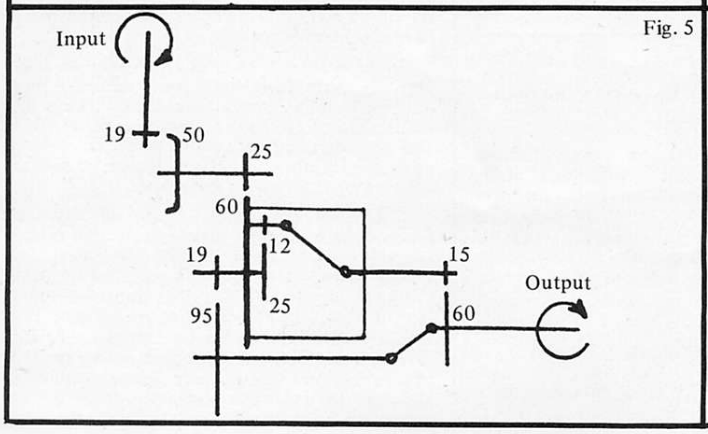


Fig. 5

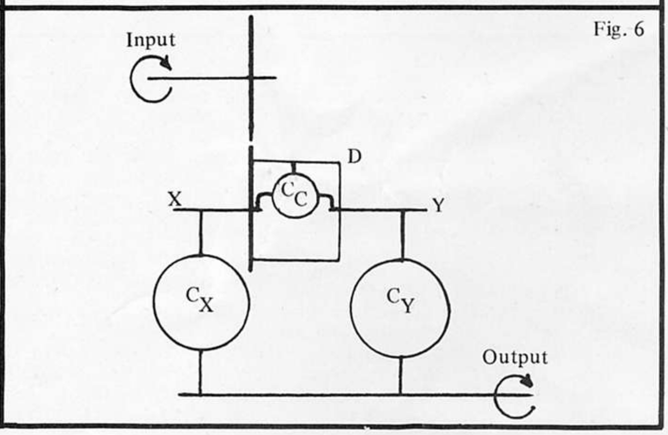


Fig. 6

thus giving a ratio of $-12/7$ rather than $12/7$ desired for $1/y$.

In making calculations on gear train design, it is necessary to adopt a 'Sign Convention' which identifies the direction of rotation of the various shafts involved. In this present article the rotation of a shaft is taken to be *positive* when the shaft is rotating clockwise when viewed from its right-hand end, and *negative* when it is rotating counter-clockwise. By extension of this principle a gear ratio will be given a positive (+) or negative (–) sign, depending upon whether it introduces a change in shaft rotational direction, or not.

EXAMPLE B The method, however, is not confined to dealing only with numbers such as 32 and 27 which are simply factorisable. Indeed, it is at its most powerful with relatively large primes. A much closer approximation to the reduction ratio needed for the Lunar Synodic Period than that used in Example A has long been known to be $19/120 \times 37/173 = 1 : 29.530583$ (an error of only -6×10^{-6}) and in the past some clockmakers have cut special gears for the primes 37 and 173. However, by using the $(A+B)/(Ax+By)$ method this special gear-cutting is found to be unnecessary.

i) $\dfrac{37}{173} = \dfrac{25+12}{125+48} = \dfrac{A+B}{Ax+By}$

ii) $A/B = 25/12$ which can be obtained in Meccano by the use of a 12-tooth clockwork motor pinion.

iii) $1/x = A/Ax = 25/125 = 1/5$

iv) $1/y = B/By = 12/48 = 1/4$

So, a layout as sketched in Figure 5 would be possible.

FURTHER DEVELOPMENT

Even further development of the $(A+B)$ $(Ax+By)$ principle is possible if there is a relaxation of the restriction to the use of only two gears within the differential-type unit. Strictly, if the two gears are replaced by a compound gear train C_C which provides a reduction ratio c between the left and right half-shafts, the overall effect becomes:

$$\frac{\text{Output speed}}{\text{Input speed}} = \frac{(1/c)-1}{(x/c)-y}$$

Again, the derivation of this expression is given in the original article; when $c = -B/A$ as in the two-gear case already discussed, this new formula, of course, reduces to the now-familiar $(A+B)/(Ax+By)$.

With this development the complete layout now takes the form indicated in Figure 6: the gear train C_C may be of any appropriate form, either using universal couplings as described previously, or not, as relevant – the only fundamental requirement is that the gear train is to be carried by the rotating cage. The reduction ratio c refers to the complete train of gears linking the two half-shafts, and if a pair of numbers are defined as A and B where $1/c = -A/B$, the expression $(A+B)/(Ax+By)$ may still be used to represent the overall gear ratio of the layout, retaining the simplicity of calculation which has already been demonstrated.

* * *

APPLICATION OF THE GENERAL TECHNIQUE

EXAMPLE C Another good approximation to the Lunar Synodic Period reduction ratio is $49/1447 = 1 : 29.530612$ (an error of 23×10^{-6}). This ratio however creates a problem in that 1447 is a prime number much too large to be cut as the teeth of a single gear. In fact, even with the $(A+B)/(Ax+By)$ technique $49/1447$ is somewhat difficult to achieve; however $-49/1447$ is much easier to produce and of course gives the same rotational speed, but with the reverse rotational direction.

i) $\dfrac{-49}{1447} = \dfrac{-56+7}{1440+7} = \dfrac{A+B}{Ax+By}$

ii) $1/c = -A/B = 56/7 = 8$

iii) $1/x = A/Ax - 56/1440 = -7/180$

iv) $1/y = B/By = 7/7 = 1$ which means that the right-hand half-shaft can be used as the output shaft.

Thus the desired result can be achieved with a gear train as sketched in Figure 7. A slight refinement can still be added even so; to avoid complications in the drive to the cage a re-shuffle to the layout shown in Figure 8 makes a better practical arrangement.

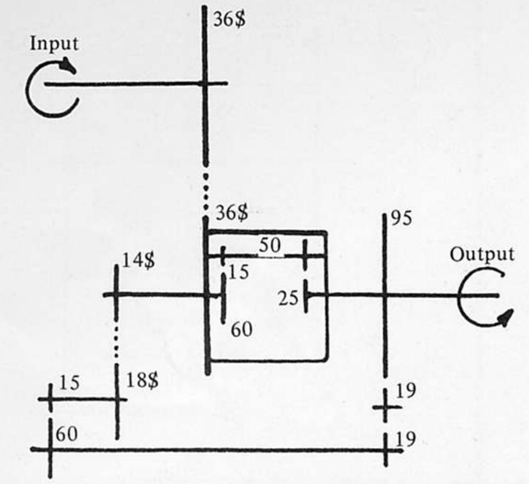
Here the left-hand half-shaft has become a Socket Coupling and drive is direct to a pair of Face Plates which are linked by Screwed Rods or other convenient means to form the cage, and then locked to the input shaft which enters through the Socket Coupling. As a result, the input and output shafts are in line with each other giving a very neat layout. The idling 19-tooth Pinion is used to ensure that the total reduction ratio in the external segment of the gear train is the $-180/7$ which is desired, with not only the magnitude of the ratio being correct, but also with the first and last shafts contra-rotating as required by the minus sign. The job is done with only eleven gears, and it is doubtful if any other arrangement would be so economical in gears, so simple, or so convenient.

TIPS FOR USERS

From the foregoing it will have been seen that the $(A+B)/(Ax+By)$ technique is an extremely powerful tool for designing gear trains – and it has the advantage that there is only one formula to remember!

All that the user has to do is to take the ratio required, split it into convenient Meccano gear multiples, and check that A/B, $1/x$, and $1/y$ can be accommodated with Meccano gears. Frequently there will be several choices avail- →

Fig. 7



able and the decision will then involve such matters as the ease of construction, or the availability of components at the time.

A useful aid is a table of 'Meccano products' of which an example is given in Table 1. This shows numbers having only prime factors of 2, 3, 5, 7 or 19, these being the factors relevant to the use of Meccano gears and sprockets. A list can then be made for the pairs of 'Meccano products' whose sum, or difference, equals the numerator or the problem ratio, and another list can be made for the denominator. These lists are examined to find pairs having simple relationships to each other. For Example B, the pairs are:

173 = 171 + 2	37 = 36 + 1
168 + 5	35 + 2
152 + 21	32 + 5
135 + 38	30 + 7
133 + 40	28 + 9
128 + 45	27 + 10
125 + 48*	25 + 12*
98 + 75	21 + 16
	19 + 18

The stars (*) show the most convenient number-pairs to use.

It is often useful to remember that a 14-tooth sprocket linkage will operate in a 1" spacing, for instance in the rotating cage, thus often taking care of otherwise awkward A/B factors, for instance:

34 = 27 + 7 = A + B can be arranged as
27/7 = 3 x 9/7 = 57/19 x 18s/14s.
37 = 28 + 9 = A + B can be arranged as
28/9 = 4 x 7/9 = 60/15 x 14s/18s.
39 = 21 + 18 = A + B can be arranged as
21/18 = 3/2 x 7/9 = 57/38 x 14s/18s.

It is also very useful to remember, as noted earlier, that if either x or y can be made equal to 1, that half-shaft can be used as the output shaft.

These are only a few of the possibilities open to the use of this technique; many similar gear trains have been built to meet specific needs, and the overall impression is that the (A+B) /(Ax+By) method is a workable system for obtaining almost any gear ratio with (probably) the minimum usage of gears.

ACKNOWLEDGEMENT

The authors of this article jointly wish to express their thanks to Patrick Briggs and Alan Partridge for the advice and encouragement given during its preparation.

TABLE 1 – 'MECCANO PRODUCTS' IN THE RANGE 1 to 1000

Numbers which do not have prime number factors other than 2, 3, 5, 7 or 19 (note that * indicates that 19 is a factor)

1	36	100	196	342*	512	735
2	38*	105	200	343	513*	750
3	40	108	210	350	525	756
4	42	112	216	360	532*	760*
5	45	114*	224	361*	540	768
6	48	120	225	375	560	784
7	49	125	228*	378	567	798*
8	50	126	240	380*	570*	800
9	54	128	243	384	576	810
10	56	133*	245	392	588	840
12	57*	135	250	399*	600	855*
14	60	140	252	400	608*	864
15	63	144	256	405	625	875
16	64	147	266*	420	630	882
18	70	150	270	432	640	896
19*	72	152*	280	441	648	900
20	75	160	285*	448	665*	912*
21	76*	162	288	450	672	931*
24	80	168	294	456*	675	945
25	81	171*	300	475*	684*	950*
27	84	175	304*	480	686	960
28	90	180	315	486	700	972
30	95*	189	320	490	720	980
32	96	190*	324	500	722*	1000
35	98	192	336	504	729	

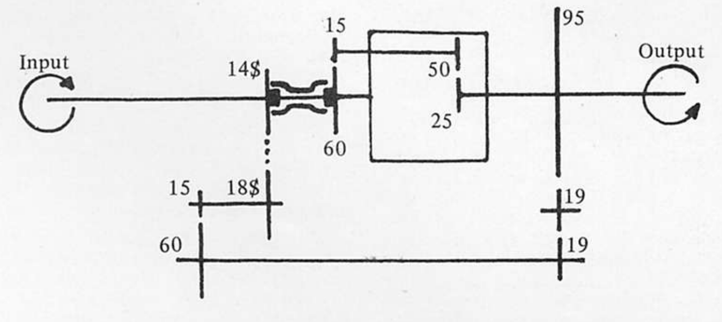


Fig. 8

MECCANO MOUSE CONTEST

Once again, this time because of the late circulation of the July MM, modellers who like a challenge were given insufficient time to tackle Alan Partridge's Meccano Mouse Contest, first announced in our April issue. For the last time, therefore, we have decided to extend the competition by a further 3 months, and *all entries must now reach us by 11th December, 1978.*

Our taskmaster has provided the following notes on the competition to set minds working, but before coming to them, we should remind readers that the task in question is to make a model which, when suitably prepared, set down on a smooth surface and released, will move along a figure-of-eight course and return to the starting point. Only standard Meccano parts (not Plastic Meccano) may be used, along with any motor made by Meccano in the last 10 years. Batteries must be carried on the model. The lightest model will win. The model itself must not be sent to us; just sufficient details to enable it to be re-built and a note of its all-up weight, including batteries. Alan Partridge now writes:–

"I have to confess that I cribbed this competition from one set a few years ago in a university engineering department. There the model had to be made from balsa, paper clips, and one standard office rubber band. I have not tried to make a Meccano Mouse myself, so most of what follows is guesswork.

"Meccano Driving Bands do not have as much stretch as most elastic bands and model aircraft rubber strip. I doubt whether they would provide enough power. The Tension Spring is very strong, but does not stretch very far; If it pulled a string wound round a rod this would probably have to be geared up to the driving axle. Spring Cord could not be twisted like rubber and Compression Springs would be awkward to use. The Magic Motor has spring power and the gearings all in one unit, and looks a good prospect. A crane motor with a few miniature heavy-duty alkaline cells, as used in flash guns, would have more than enough power for dozens of runs, but more weight. A small fly-wheel set in Elektrikit needle bearings would be worth trying; it has been shown that a high speed fly-wheel could run a city car for over 10 miles.

"The other problems to be solved are the form of the vehicle and how it is to be steered on a figure-of-eight course. If it starts at the centre, it needs to do one complete circle to left and one complete circle to right, or vice versa. The lightest form is likely to be a three-wheeler with one wheel steered. We cannot afford the weight of a differential, so only one of the others will be driven and the third will be loose. If the steered wheel has a tiller, this can rest against a cam, kept in contact by a Compression Spring. The cam can be driven with gearing-down, or the equivalent in Driving Bands or string, from the axle of the driving wheel; or more directly from the winding shaft of a Magic Motor. Alternatively the steering wheel can be set hard over to the opposite side, a catch being released by string taken up on an axle or winding shaft.

"Another interesting possibility is rather like a child's bicycle with stabilizers, but the two small wheels and the larger central wheel are all fixed on one driving axle. The front, or rear, single wheel castors revolve freely. The vehicle circles in whichever direction it is leaning, and a weight must move across to tip it over at half time. This links up with another possible source of power: a falling weight.

"Two aluminium rods from a Clock Kit pendulum would form a light triangle with a pulley at the top; the string from a falling weight would pass over the pulley and round the driving axle. Then, if the weight slid down one rod to one side, this could also tip the vehicle sideways as required.

"What else haven't I thought of, and which will be the lightests? I'll be very interested to see."

ALAN PARTRIDGE