A Meccano Planimeter

Determining Areas of Drawings by Mechanical Means

By H. F. Lane

A PLANIMETER is an instrument for determining by mechanical means the area of any figure, whether regular or irregular. Given certain principal dimensions, the area of a regular figure, such as a square, circle, triangle, etc., may readily be calculated from formulæ. There are other figures less regular than these for the graph, or outline, of which, an algebraic equation can be determined, and the area obtained by integration. But beyond these again are completely irregular figures, such as those shown in Figs. 1 and 2. Should the area of such a figure be required, it can be determined in the following three ways :—

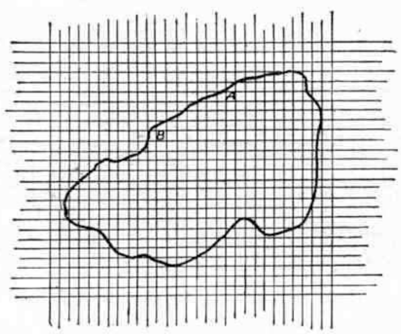

Fig. 1. Graphical method of measuring areas

(1) Graphical. Either by drawing the lines in on the paper or by placing on top of the paper a transparent sheet so ruled, we cover the figure with a multitude of tiny equal squares, the area of each square being some definite fraction of the scale in which the result is required, and then count up the squares (see Fig. 1). No error need result so long as we are dealing in whole squares, but when we come to the outer edges of the figure we shall find some of the squares incomplete, because cut off by the boundary line of the area (Fig. 3). Here each partial square must be considered on its own merits, and a search made round the edge of the figure for a second partial square of such a shape that the portion inside the area of the second square, added to the portion inside the area of the first square, will together equal a complete square, and so be included in our computation of the area. For example, the small portion B added to A makes a complete square.

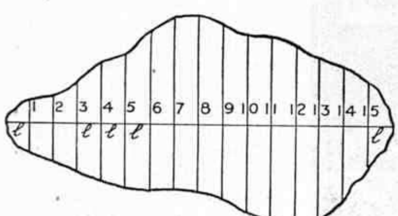

Fig. 2. Simpson's Rule

To avoid counting a square twice or omitting it altogether, it is usual to shade, dot, or mark in some fashion, each square or partial square as we include it. This method of obtaining the area of the figure is very laborious, and the accuracy of the result obtained depends, obviously, on our "eye" for estimating the various pairs of complementary portions to complete the broken squares.

(2) By Calculation. In this method we find the length of the figure by drawing inside the area, and bounded by the edges of the area, the longest straight line possible (Fig. 2). We then divide this line into an even number of small equal parts, and draw through each division a line at right angles to the length of the figure. The additional lines so obtained are called "ordinates" and again are bounded by the edges of the area. Let l be the width of each small division, i.e., the distance separating each pair of ordinates. Add together the lengths of the two extreme, or outside, ordinates (1 and 15), and call the result A. Add together the remaining odd ordinates (3, 5, 7, etc.) and call the result B, and then the even ordinates (2, 4, 6, etc.), calling the result C. Then, supposing we have measured the various lengths in inches and decimals of an inch, we have :—Area of figure in square inches =

$$(A + 2b + 4c)\frac{l}{3}$$

This is known as Simpson's Rule, and again must be regarded as an approximation only, because if we have not a sufficient number of ordinates, we do not allow sufficiently for the irregularities of the figure, because each ordinate will have too large a relative value. On the other hand, although theoretically the equation only ceases to be an approximation and becomes perfectly true when the number is infinite, the more we increase the number of ordinates and decrease l, the more do we introduce personal error due to faulty measurement.

(3) By Mechanical Means. By the aid of an instrument termed the planimeter, the result can be arrived at with more accuracy and very much more rapidly, since all we have to do is to guide the pointer of the instrument carefully round the boundary line of the area, and read off the result on the recording apparatus.

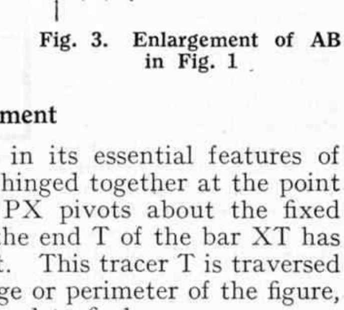

Fig. 3. Enlargement of AB in Fig. 1

There are several types of instrument designed for this purpose, some of them extremely complicated and able to perform a variety of functions, but the one in common use is Amsler's Polar Planimeter. It is on this principle that the Meccano model described below has been constructed.

The Theory of the Instrument

The instrument consists in its essential features of two bars PX, XT, (Fig. 5) hinged together at the point X. One end of the arm PX pivots about the fixed point P in the paper, and the end T of the bar XT has a tracing point secured to it. This tracer T is traversed round the entire outside edge or perimeter of the figure, the area of which it is desired to find.

The bar XT carries a wheel W that is sometimes rotated by the motion of T (when motion is perpendicular to XT), sometimes slid sideways without revolving (when motion is parallel to XT), and at other times undergoing a combination of both movements (at any other angle). As the wheel undergoes these various movements, it is always turning first in one direction and then in the other. When it has completed the whole perimeter of the figure, however, and returned to the point at which it started, it will be found that the wheel has rolled more in one direction than the other, and it is this difference in travel that measures the area of the figure, according to some predetermined scale.

It is not proposed to go too deeply into the mathematical considerations of the planimeter, but in the simple figure MNOQ (Fig. 5) it may readily be seen that the wheel records a definite sector MNX whilst rolling from M to N with X remaining stationary. The area of MNX is dependent entirely on the arc MN—since XM and XN are fixed lengths—and may therefore be recorded by the wheel. In this broad and approximate demonstration of the principle of the planimeter, we will disregard the slight roll of the wheel in moving from N to O. This roll of the wheel is very much less than the distance NN_2 (obtained by drawing ON_2 parallel to QM), because the point X takes up a new position X_2 as the bar XT moves from N to O, which counteracts the effect of the distance NN_2.

In moving from O to Q, we roll back again a certain amount of the roll of the wheel already recorded, but as in this figure the arc OQ is shorter than the arc MN, a diminished reading still remains. Also disregarding the second slight roll as the tracer moves from Q to M and still speaking approximately, we have recorded the area MNX less the area OQX_2, and the difference between the two is the required area MNOQ.

It must again be insisted that the above explanation is a non-mathematical approximation; the small rolls occurring while moving from N to O and from Q to M do affect the result and it is because they do so and further entail a movement of X, that the instrument is accurate for any figure, however irregular.

It will be noted that the point P lies outside the area. It is not proposed to mention the added complications entailed when the figure is so large that P must be placed inside the area.

The Construction of the Model

As will be seen from Fig. 4 the construction of the model is very simple. The point P in Fig. 5 is represented by a 3½" Rod 1 that is held in a Bush Wheel screwed to a Designing Machine Table (part No. 107). The bar 2 is retained in position on the Rod 1 by means of Collars. The bar 3 consists of four 18½" Angle Girders bolted at one end to a Channel Bearing. The tracer, which consists of a 3½" Rod filed to a sharp point, is held in the boss of a Fork Piece secured to the upper pair of Angle Girders, and its lower end is passed through the centre hole of a Double Bracket that is secured to the lower pair of Girders. The two pairs of Angle Girders are spaced the correct distance apart at this end by 1½" Strips.

On the ½" side of the Channel Bearing, 2½"×1" and 2½"×1½" Double Angle Strips are bolted as indicated in the illustration, the former being used to connect the bar 2 to the tracing arm 3.

The wheel 6 consists of a 57-teeth Gear and a Flanged Wheel secured together on a 3" Rod that is journalled freely in the bottom hole of the Channel Bearing and in the end of the 2½"×1½" Double Angle Strip. A strip of paper is stuck round the circumference of the Flanged Wheel, and this Strip is marked off into 10 equal main divisions, each of which is further divided into ten subdivisions, making in all 100 divisions of the circumference of the Flanged Wheel. The main divisions should be numbered 0 to 9. The 1½" Strip 7 serves as a sighting piece with which to read the graduations on the Flanged Wheel, the graduations being viewed through the bottom hole of the Strip. In order to lessen the possibility of error a fine silk thread, termed a "spider line," should be glued across the hole in the Strip 7 so as to indicate the exact

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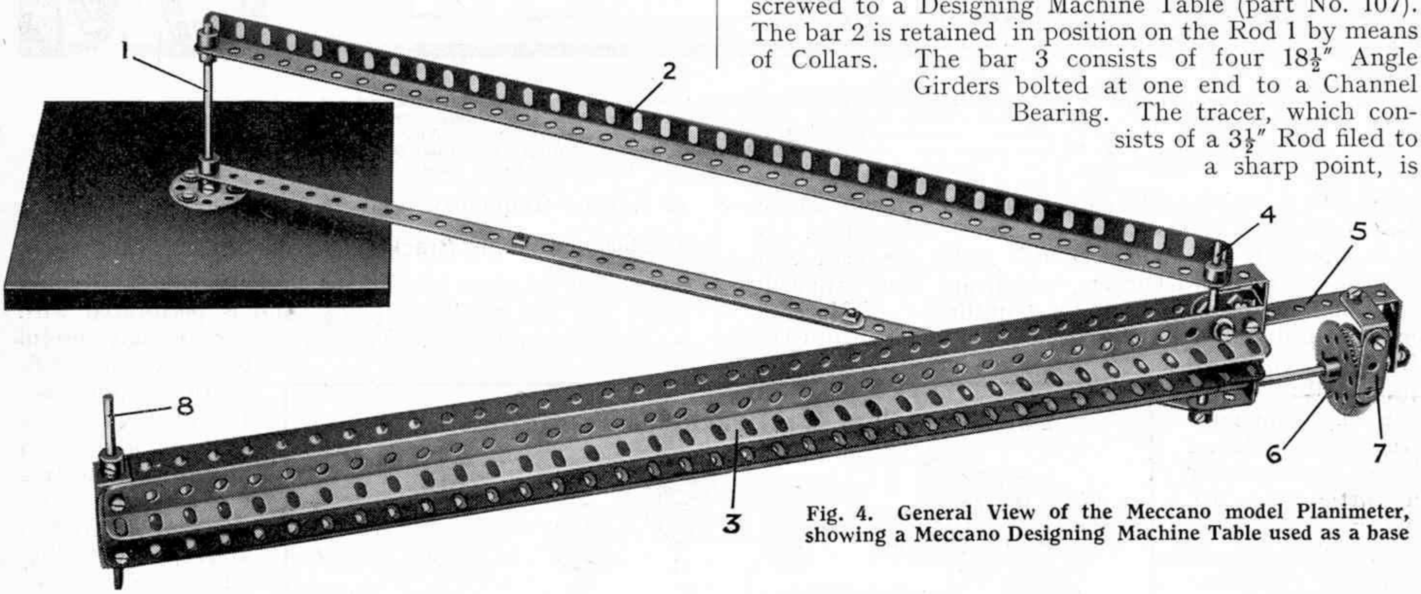

Fig. 4. General View of the Meccano model Planimeter, showing a Meccano Designing Machine Table used as a base

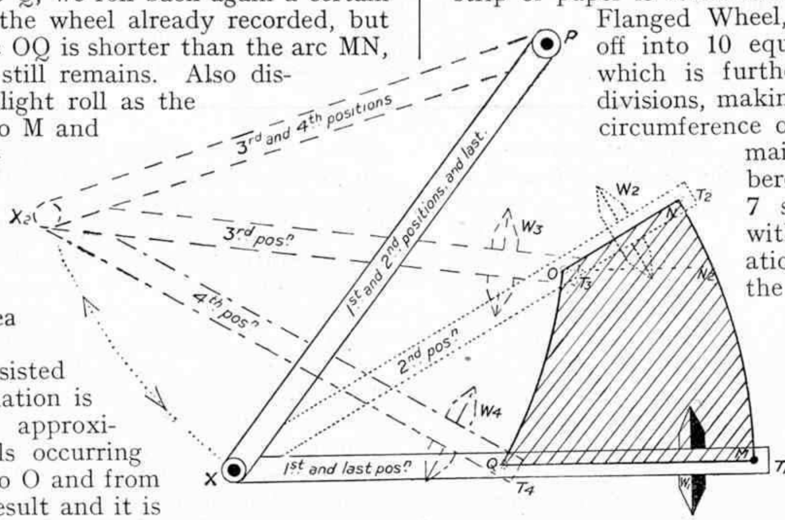

Fig. 5. Drawing illustrating the theory of the instrument